

LT<sup>2</sup>

# Learning and Teaching with Learning Trajectories, Birth to Grade 3: General Introduction

FACILITATOR GUIDE



Marsico Institute



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## Introduction to the Learning and Teaching with Learning Trajectories Guides

The *Learning and Teaching with Learning Trajectories Facilitator Guides* (aka *Guides*) are intended for use by facilitators, leaders, and professional development experts to work with teachers and caregivers, birth to grade 3, on mathematics education.

This first *General Introduction Guide* provides general preparations and tips for organizing and beginning this work using the other, content-oriented Guides. It also introduces the Learning and Teaching with Learning Trajectories [LT]<sup>2</sup> tool and provides one activity you can use to start using the tool with your teachers.

## Facilitator Guide Organization

All other Guides are organized by topics and grade levels, and then sequenced by learning trajectory levels on [LT]<sup>2</sup>. Each session includes ways to support teachers in learning the trajectories and instructional activities to support children's development. Each session prompts facilitators to play videos from the [LT]<sup>2</sup> website. Facilitators should familiarize themselves with the website to have an in-depth working knowledge of the resource. Facilitators should make sure computers or tablets are easily accessible for participants.

**Developmental Progression**

Search for activities and games

PRE-SCHOOL

2 Yrs 3 Yrs 4 Yrs 5 Yrs **K** 1st 2nd 3rd

Highlight typical levels for age / grade

Filter trajectories by category

- Number
- Operations & Algebraic Thinking
- Geometry
- Measurement and Data

**Counting**

- Pre-Counter
- Chanter
- Reciter

**Subitizing**

- Pre-Explicit Number
- Small Collection Namer
- Maker of Small Collections

**Comparing Number**

- Many-to-One Corresponder
- One-to-One Corresponder
- Object Corresponder

## Goals

Over the course of the year, we will learn about how to use the following:

- Learning trajectories for each major mathematics topic;
- Using learning trajectories for observation and other authentic assessment strategies;
- Supporting mathematical development in the classroom;
- Recognizing and supporting math throughout the day;
- Setting up math learning centers teaching with computers (including use of the management system and research-based teaching strategies);
- Teaching small-group activities supporting mathematical development in the home; and
- WaKIDS, the Common Core State Standards and assessments;

## An Introduction to Learning Trajectories

The guides are, of course, all about learning trajectories. What are they? To begin, we know that children's thinking follows natural developmental paths in learning math. When teachers understand these paths, and offer activities based on children's progress along them, they build math-learning environments that are developmentally appropriate and particularly effective. A useful tool in understanding and supporting the development of children's mathematical reasoning is a math-learning trajectory. There are learning trajectories for mathematics at all age levels, from birth throughout the K-12 school years.

Math learning trajectories have three parts: a mathematical **goal**, a **developmental path** along which children's math knowledge grows to reach that goal, and a set of **instructional tasks**, or activities, for each level of children's understanding along that path to help them become proficient in that level before moving on to the next level. Let's examine each of these three parts.

**Goal.** The first part of a learning trajectory is the goal. Goals should include the big ideas of math, such as "numbers can be used to tell us how many, describe order, and measure" and "geometry can be used to understand and to represent the objects, directions, and locations in our world, and the relationship between them" (Clements, Sarama, & DiBiase, 2004).

**Developmental progression.** The second part of a learning trajectory consists of levels of thinking, each more sophisticated than the last, leading to achieving the mathematical goal. That is, the developmental path describes a typical learning route children follow in developing understanding of and skill in a particular mathematics topic.

Learning trajectories are important because young children's ideas and their interpretations of situations are different from those of adults. Teachers must interpret what the child is doing and thinking and attempt to see the situation from the child's viewpoint. Knowledge of developmental paths enhances teachers' understanding of children's thinking, helping teachers assess children's level of understanding and offer instructional activities at that level. Similarly, effective teachers consider the instructional tasks from the child's perspective.

**Instructional tasks.** The third part of a learning trajectory consists of sets of instructional tasks or activities matched to each level of thinking in a developmental progression. The tasks are designed to help children learn the ideas and practice the skills needed to master that level. Teachers use instructional tasks to promote children's growth from one level to the next.

The learning trajectories approach to instruction "is to find the mathematics in, and develop mathematics from children's experiences and interests" See the attached article (Brown, Sarama, & Clements, 2007) for illustrations of how teachers use learning trajectories for instruction.

The learning trajectories are designed to develop teachers' content knowledge by explicating the mathematical concepts, principles, and processes involved in each level and the relationships across levels and topics. For example, they introduce the components of geometric shapes (e.g., correct definition of "side") as well as relationships between components (e.g., sides forming a right angle) and shape classes (e.g., a square as a subcategory of rectangle and justification for this based on properties). The learning trajectories are intended to develop teachers' knowledge of students' developmental progressions in learning that content (moving from intuitively recognizing shapes as unanalyzed visual wholes, to recognizing components of shapes, to hierarchically classifying shape categories). They are designed to develop teachers' knowledge of the instructional activities designed to teach children the content and processes defining the level of thinking in

those progressions and to inform teachers of the rationale for the instructional design of each (e.g., why certain length sticks are provided to children with the challenge to build specific shapes). The learning trajectories assist curriculum enactment with fidelity in that they connect the developmental progressions to the instructional tasks, providing multiple guidelines or sources of stability in teachers' instantiation of the instructional activities (cf. Ball & Cohen, 1996). Finally, learning trajectories are designed to motivate and support the use of formative assessment.

### **Professional Development with Learning Trajectories**

We need to ensure that training addresses each of the three components of the learning trajectories.

- To understand the *goals*, teachers must learn core mathematics concepts and procedures for each topic. For example, they will discuss the system of verbal counting based on cycling through 10 digits and the concept of place value.
- To understand the *developmental progressions* of levels of thinking, teachers will view video segments that illustrate each level and discuss the mental "actions on objects" that constitute the cognitive components of each level.
- To understand the *instructional tasks*, teachers will study the tasks or activities in the curriculum, practice enacting them with each other, and also view, analyze, and discuss video of the enactments in classrooms.

Teachers need help learning how to use the learning trajectories as a basis for formative assessment, a key to high-quality teaching. Teachers need to discuss and practice how to interpret children's thinking and select appropriate instructional tasks for the class (e.g., compacting the curriculum if most children can learn it at a faster pace) and for individuals.



## The Learning and Teaching with Learning Trajectories [LT]<sup>2</sup> Tool

[LT]<sup>2</sup> is a web-based tool for early childhood educators to learn about how children think and learn about mathematics and how to teach mathematics to young children (birth to age 8). The website provides teachers with access to information about *Learning Trajectories* for math. Teachers can also review short video clips of children's thinking along the math *Learning Trajectories*. Users can access hundreds of classroom activity ideas to support children's development along the math trajectories.

Math learning trajectories have three parts: a **mathematical goal**, a **developmental path** along which children's math knowledge grows to reach that goal, and a set of **instructional tasks**, or activities, for each level of children's understanding along that path to help them become proficient in that level before moving on to the next level. Users will explore these three components for each learning trajectory.

*A complete manual is available in a separate document, "Learning and Teaching with Learning Trajectories [LT]<sup>2</sup>—User Instructions."*

Computer screen icons appear throughout the Guidelines directing facilitators to play videos from the [LT]<sup>2</sup> website. Facilitators should familiarize themselves with the website to have an in-depth working knowledge of the resource. Facilitators should make sure this website is loaded on computers and easily accessible for presentation when the time comes.

To log on to the [LT]<sup>2</sup> website go to [www.LearningTrajectories.org](http://www.LearningTrajectories.org)

# Planning for the Sessions

## Make This All Your Own!

The guides are not meant to be rigid instructions. They are to be used as practical resources and purposeful references for facilitators who have a sound understanding of early mathematics learning trajectories. The training will be more effective, and you will be more comfortable, if you bring your own unique style and experiences. In other words, “Make it your own.”

Consider the following:

- The teachers you will be working with will bring diverse experiences, from those experienced working with young children to those who are completing their first year of teaching. There may be lead teachers, assistant teachers, and paraprofessionals. It is important to find ways to engage all participants and use their strengths to enhance the ideas outlined in the workshop.
- The educators you are working with will have varying degrees of comfort and knowledge with the ideas and documents and resources that describe them. The more familiar you are with these documents and resources, the better able you will be to help participants connect their experiences and learning to them.
- Throughout the session, look for opportunities to highlight the expertise of experienced educators, and encourage them to share their knowledge with others. For example, you may listen for particularly insightful comments during small group discussions and ask the participant to share with the whole group.
- A variety of techniques representing the different ways that people learn have been incorporated into the training. They include mini-lectures, videos and information on [LT]<sup>2</sup>, “think-pair-share” interactions, recording on chart paper, and large and small group work. Feel free to adapt these activities, taking into consideration the size of the group, the venue, and the experience levels of the participants. Consider the expertise of participants (e.g., including someone experienced working with a child with a disability during discussions about differentiating activities for children with special needs).
- *Do not feel the need to do every activity. These are illustrations. For the sake of time, or interest, use only those you believe will help the participants.*

## Setting Up

For many facilitators, these suggestions will be well-known, but perhaps useful reminders. Prepare a welcome sign on chart paper. Arrange participant handbooks, markers, chart paper, post-its, and pens in a way that makes sense for the space. You may want to add additional elements to create a welcoming space (e.g., music, children’s books on transitions, chart paper with agenda, etc.). Create a chart titled “Parking Lot” for questions that arise during the session which are not relevant to the topic.

## Training Materials

Prior to the training facilitators should familiarize themselves with the manipulatives to be used by participants. Facilitators should be comfortable identifying appropriate manipulatives when leaders have questions and should be able to demonstrate appropriate use of materials as needed.

**All participants (leaders) must have the following during training:**

- Activities (from [LT]<sup>2</sup>) identified for practice in the *Guides*.
  - Participants must have a copy of each pdf document if the activities as you have a projector and so forth to viewing the videos with the whole group *or*
  - Participants must be on computers (paired up is fine) for downloading the documents and viewing the videos.
- Access to manipulatives.

**Facilitators are responsible for ensuring materials and technology components are ready.**

## Suggestions for Co-Facilitation

In the event that you are co-facilitating your session please keep in mind the following practices. Co-facilitation allows one person to present while the other observes and supports their partner. Partners should divide the material in a way that lets them capitalize on individual strengths and have their own moments to lead discussion. Remember that you are modeling co-teaching practices for the participants.

### ***Before the Training***

- Schedule time for planning together
- Discuss each other's style of planning and facilitating
- Take time to discuss your views about the training materials
- Especially examine areas of disagreement
- Discuss any concerns about potential challenges that participants may present
- Find out whether and when it is okay to interrupt
- Decide how to keep track of time
- Plan ways to give signals to one another
- Divide facilitation of activities fairly
- Share responsibility equally in preparing and bringing training materials and resources
- Agree to arrive at the training site in time to set up and check-in before the training begins

### ***During the Training***

- Keep communicating with each other throughout the training
- During activities that don't require constant attention, check-in with one another
- Include your co-facilitator even when you are leading an exercise or discussion, by asking, for example: "Do you have anything to add?"
- Two facilitators can manage a group better than one. The second person can help gauge participants' reactions and notice whether people seem to be on-track.
- Co-facilitators can also help hand out materials, assist in monitoring discussions and/or coach participants in groups.
- Co-facilitators can monitor and handle problems with the physical environment, latecomers, phone calls, audio-visuals, and other logistical matters.
- Use lots of eye contact

# Guidelines for Conducting Sessions

## Welcome

Welcome the group warmly and introduce yourself.

## Organizing Small Groups

Let participants know that they will be working in small groups. The groups should be made up of about 4-5 participants.

Ask participants to form small groups and assign each person in the small group a focus or role such as leader or recorder.

You may wish to have each small group focus on the needs of a specific group of children:

- Children whose native language is not English
- Children with an Individualized Education Plan (IEP)
- Children who need targeted support

Explain that we are forming groups that reflect the diverse needs of children in classrooms and assigning roles to participants so that the needs of each group are considered and included in discussions and planning. Describe each category to the participants and recommend that participants take a role with which they have expertise, experience, or interest.

## Starting the First Session: Introducing Learning Trajectories and the [LT]<sup>2</sup> Tool

|  |  |
|--|--|
| <b>Objectives</b> <ul style="list-style-type: none"> <li>• Understand the importance of early math in children’s development.</li> <li>• Understand the components of a Learning Trajectory.</li> <li>• Be able to navigate the [LT]<sup>2</sup> website.</li> </ul>   |  |
| <b>Facilitation Materials</b> <b>Directions</b>  |  |
| <p>[LT]<sup>2</sup> User Instructions</p> <p>Access to the Internet</p> <p>Learning Trajectories overview document</p>   | <p><b><u>Introduce Learning Trajectories</u></b></p> <ol style="list-style-type: none"> <li>1. Ask participants what they have heard about learning trajectories.</li> <li>2. Play the introduction to <i>Learning Trajectories</i>: <a href="http://www.learningtrajectories.org">www.learningtrajectories.org</a></li> </ol> <p><b><u>Discuss:</u></b></p> <p>What are the primary components of a <i>Learning Trajectory</i>?</p> <p style="padding-left: 40px;">The goal, the developmental levels and the instruction to help children get to the next level to meet the goal.</p> <p>How do <i>Learning Trajectories</i> help guide students toward the mathematical goal in these levels?</p> <p style="padding-left: 40px;">The instruction is key. An LT approach is not about targeting a child at a level and periodically reassessing. The instructional activities are intended to be the “just right” instruction children need.</p> <p>How can the <i>Learning Trajectories</i> help you in your planning and teaching?</p> <p style="padding-left: 40px;">Answers will vary, but participants should know that this is a developmental perspective, that learning more about how children learn mathematics will enable them to better see children’s strengths, meet children where they are, and differentiate appropriately to help all children meet the goals.</p> <p>How does filtering the age or category change the highlighted content on the page?</p> <p style="padding-left: 40px;">Explore [LT]<sup>2</sup> with users with the guiding questions to the right and using the background information below.</p> |
| <b>Background Information</b>  |  |
| <p>When exploring the developmental progression page, remember a level is highlighted when a specific age or a category is selected, such as number or geometry. A teacher can select a level that is not highlighted and still access the content within that level. The highlighted sections merely give the levels that are coded for the specific age.</p> <p>When starting with the website for the first time, it is useful to explore the developmental progressions, starting with the early levels and moving up to more advanced levels.</p> <p>Videos show children’s development. Note that there are multiple videos for some levels and that these are most often interview situations.</p> <p>Instructional activities here are designed to help students reach that level. The thumbnails with the play button indicate a video is available.</p> <p>Talk with teachers about positive adaptations and lethal mutations when adjusting for their classroom contexts.</p> |  |